

Integrated Technology in Undergraduate Mathematics: Issues of Assessment

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Abstract: *Effective integration of technology into the teaching and learning of mathematics presents a significant challenge to tertiary mathematics educators. Assessment issues are widely considered in the literature as one critical factor in technology implementation, and this was confirmed in a PhD study investigating the overall use of technology in undergraduate mathematics. This paper expands on the findings reported at the 2009 ATCM conference in Beijing. It first introduces the taxonomy developed to describe and compare technology use within individual courses and departments. The taxonomy identifies six overarching characteristics in the construction of an Integrated Technology Mathematics Curriculum (ITMC). The paper then specifically examines the Assessment characteristic of the taxonomy, using evidence gathered from an observational study of technology implementation at The University of Auckland. The findings of this study suggest that aspects of assessment such as curricular congruency, equity and the advantages and affordances provided by different technologies require continued attention and constant vigilance, if integrated technology is to be successfully implemented and sustained.*

1. Introduction

An earlier study by Oates [1] identified the considerable variety of ways in which integrated technology is interpreted in the literature, and concluded that a clearer means of characterising what is meant by *integrated technology* was required. Responses from an exploratory survey of undergraduate mathematics lecturers were used to propose an initial model for describing integrated technology. A strategy was developed to describe and compare technology integration for different courses and institutions using this model [1, pp. 286; 289].

Building on the results of this initial study, a wider international survey of undergraduate mathematics educators was then conducted, seeking to identify the essential characteristics of a tertiary *integrated technology mathematics curriculum* (ITMC) [2]. This second survey aimed to investigate more closely a number of factors identified in the pilot study and the literature, particularly those associated with the use of technology at the tertiary level. These include student instrumentation [3, 4]; and the affordances, constraints and obstacles encountered with different technologies [5, 6]. Other factors considered were the effect of research mathematicians' beliefs about mathematical knowledge, technology and pedagogy on their use of technology [7, 8]; and the relationship between mathematicians' experience with different technologies within their own research domains, and their pedagogical technical knowledge (PTK). PTK is characterised as the necessary knowledge of the principles and techniques required to teach mathematics using a given technology [9]. This paper reports briefly on the taxonomy of integrated technology developed in the wider study, and then examines more closely one of the six characteristics described in the taxonomy in closer detail, namely assessment. The observational study of technology implementation in undergraduate courses at The University of Auckland, carried out as part of the wider study over the period 2001 to 2008, identified assessment issues as one of several critical elements of the taxonomy affecting the comparative success of such technology implementations.

2. The Study

A full description of the overall methodology of this study, including the construction and administration of the survey used to develop the taxonomy of integrated technology is provided by Oates [2]. He describes how questions were refined from the issues initially identified in Oates [1], and later amplified by a more extensive review of the literature. This review yielded questions such as those about changes to the relative epistemic, pedagogical and pragmatic values of curriculum topics when using computer algebra systems (CAS) [3, 10, 11]; questions seeking mathematicians' beliefs about the comparative values of calculators and computers [12]; and specific questions about assessment issues such as access to technology in tests and examinations, characterised as curricular congruency by Leigh-Lancaster [13], and the nature of questions asked in such formal assessments [14, 15, 16, 17]. Sample questions from the survey, with respect to curriculum development, assessment, and beliefs about technology, are shown in Table 1.

Table 1 Sample Survey Questions [2]

Survey Section	Sample Question
Part B: Nature of Technology Usage.	<p>B2 (i) Has there been any review of your curriculum that considered the way in which technology will be used or impacts on the course(s)? Please specify, e.g. no specific considerations of technology; we have considered technology in the process of our normal course revision/development; we designed a new course incorporating technology.</p> <p>(ii) Have there been any changes to the content of the course because of the technology you use? Please specify, e.g. have some topics been dropped? Have any new topics been added? Has the order of topics been changed?</p> <p>(iii) Do you use new textbooks that were chosen at least partly because of technology, or have you written specific technology sections in course-books?</p> <p>B3 (i) Have there been any changes in assessment because of technology, e.g. assignment questions that require technology; laboratory project?</p> <p>(ii) Is technology permitted, required or not allowed in assignments, tests or examinations? Please specify, e.g. allowed or required in assignments; not used in tests/exams as not in laboratory; can use in exam (e.g. CAS or graphics calculator); question in exam that requires knowledge of technology (e.g. Matlab printout); tests or exams held in laboratory.</p>
Part C: Beliefs about technology usage	<p>C1 (iii) To what extent do you believe technology should be integrated into undergraduate mathematics courses? Please specify, e.g. it should be available and used at all appropriate opportunities including in class, assignments, tests, exams, homework; should be some elements of "technology-free" assessment in the course; should be limited to specific examples where it is useful, chosen by teaching staff (give example if possible).</p>

The survey was sent to 134 colleagues from 44 tertiary institutions involved in the teaching of undergraduate mathematics. Given the complex detail of responses required by the survey, the achieved response rate of 42% was highly satisfying, although Oates [2, p. 192] does investigate any issues that may have arisen from the level of non-response. However, he largely considers these to be of minimal significance, observing for example that “ it is entirely reasonable to assume that most of the non-response was due to lack of time and the demanding nature of the survey, rather than any significant difference in the views of respondents with respect to technology” [2, p. 192]. Respondents represented 72 different undergraduate courses, from 31 tertiary institutions in 8 countries (Australia, Canada, France, New Zealand, South Africa, United Kingdom, United States, and Uruguay). Responses to the survey were compared initially against the model developed earlier [1], documenting any responses that were problematic to locate within the existing coding framework. Oates [2] notes that a considerable number of such responses necessitated substantial changes to the original coding system, and a corresponding refinement of the taxonomy. Four respondents from the survey were asked to review the subsequent classification of their responses, as a check on the degree to which they agreed with the assigned categories.

The complete taxonomy is not reproduced here, but a summary of the six major components is provided in Table 2, with some exemplars from the survey to illustrate the focus for each of these.

Table 2 A Taxonomy for Integrated Technology

Taxonomy Component	Characteristic Survey Response for Taxonomy Component
Access	“It has many benefits if all the students can reach almost the same technology; otherwise it creates important differences between them. I would like to see all my students using laptops, as in the private universities.” (Uruguay)
Assessment	“Students may use any hand held calculator, but in exams they must show full written working to reach the answer. Calculators are often used to check results”. (Australia)
Organisational Factors	“Bureaucracy slow to change. Use often isolated to single course.” (South Africa)
Mathematical Factors	“Less emphasis on techniques, more powerful visualisation.” (New Zealand)
Staff Factors	“Technology should be integrated only by staff who believe it is useful. Imposition of technology seems to have a negative effect on all involved.” (Australia)
Student Factors	“It’s difficult (for students) to make sense of the use of technology, especially those who had High School maths teachers with strong opinions against the use of technology.” (Canada)

The full taxonomy [2, pp. 205-206] describes a complex range of inter-related factors that should be considered within each of the six main components depicted in Table 2, along with studies from the literature that reference each. Those for the *Assessment* component are provided next (see Table 3), when assessment issues are considered in more detail. Oates [2, p. 207] notes that the complexity of issues identified in the taxonomy makes it extremely difficult to encapsulate concisely the characteristics of integrated technology, as was done for example with the radar diagrams used to compare technology use between different courses in the earlier study [1]. He emphasises that one of the more significant findings of the later study lay in the interdependency observed between the elements of the taxonomy. The results suggest that it is essential to recognise

the inter-related structure of the taxonomy. He concludes that addressing the factors in a comprehensive fashion leads to higher and more sustainable levels of technology integration, observing that “...while attendance to some elements in isolation may obviously stimulate changes, it is difficult to achieve effective, sustainable technology integration through such a limited approach” [2, p. 252].

However, despite the complexity and inherent inter-dependence of the taxonomy elements, the radar diagrams developed in [1] still provide an effective visual comparison of overall technology integration, useful for example in bench-marking between courses, or to make the case for greater levels of integration in specific departments or individual courses. It could also show the progression of technology integration over time, and help to identify specific areas within the curriculum where attention to technology issues may be needed [2, p. 207]. An example of technology use in one particular course is shown in Figure 1 below. The extent of technology integration for each radial on the diagram is measured on a 5-point scale (low to high), estimated by the researcher in similar fashion to that described in the earlier study [1]. Despite the estimated nature of the measurement, the diagram still helps to identify the lack of organisational support for technology in an otherwise integrated-technology course. The respondent for the course depicted in this diagram felt frustrated by the wider department’s lack of interest in technology, noting that “since I am almost the only instructor in our department who uses technology consistently, I have to use the same book as is used in other sections of the course” [2, p. 207]. The assessment radial also falls short of a fully integrated value of five, since the students in this course are required to sit the same examination as other students in the department, without access to their usual technology.

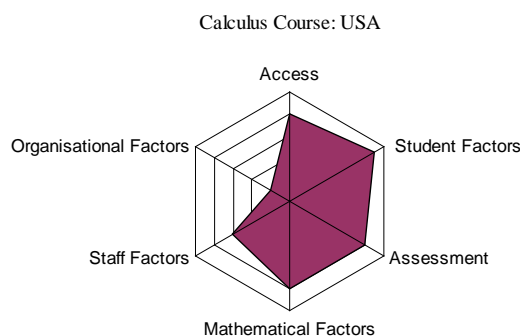


Figure 1 Calculus course from the United States showing gap in technology integration for “Organisational Factors”.

The impression gained from Figure 1 should be sufficient to stimulate further investigation into the underlying causes and effects of the organisational issues identified as under-attended. Here, the effects are the frustrations observed for some staff as in the earlier quote, and inconsistencies in technology use between courses, which resulted in complaints from students. Possible causes for the technology position that led to these effects can be seen later in the same response:

The department has discussed the possibility of introducing technology in its pre-calculus courses, although the last official discussion was about 10 years ago. The final decision has always been to maintain the status quo; i.e. no technology. The two reasons which seem to carry weight are: (a) our pre-calculus courses are precisely that: preparation for calculus; and students need to improve their (pencil-and-paper) algebraic skills in order to succeed at calculus; (b) purchase of technology, such as a graphing calculator or student copy of Maple, would be an additional financial burden on many of our students (USA).

3. Assessment Issues

The full taxonomy detailed in Oates [2, pp. 204-205] identifies five main issues with respect to technology and assessment, each of which is considered widely but often individually in the literature. These are summarised in Table 3 below:

Table 3 Sub-Categories of the *Assessment* Component from the Taxonomy in Table 2

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1. Congruency between pedagogy and assessment: Consistency of technology use in classes, assignments, homework, tests, exams;
 2. The nature of questions asked in summative assessments (e.g. tests, examinations): Level of difficulty; Technology Assumed? Neutral? Active? Free? Prohibited?
 3. Fairness: Equity of student access to, and experience with, the use of technology (instrumentation, constraints, affordances and obstacles);
 4. The opportunity for alternative forms of assessment: e.g. Computer-based assignments; on-line testing, submission; Computer-aided testing; Instructor PTK;
 5. Recognising different student solutions.
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One of the survey responses indicative of the five issues outlined in Table 3 was included previously in Table 2, other responses that demonstrate further aspects of these issues include:

“The traditional three-hour written exam has been with us for decades if not centuries. Computers will become increasingly better for the testing of key mastery skills in all subjects. They are complementary...A computerised formal test or exam can impose an additional stress on students and must only be introduced when students are fully experienced in its use” (United Kingdom).

“Fully integrated technology in class/assessment/exam. Chosen by student. Part of learning is to use and choose appropriate tools” (Australia).

“I think that any mathematics course should have elements of “technology-free” assessment. There is no substitute to human contact and human thinking. Critical thinking and ingenuity will still be important even when computers could eventually exhibit “common sense”. For this reason keeping some of the mathematics teaching “technology free” is important” (New Zealand).

“Should be used where it can enhance understanding, but limited elsewhere. Maths is after all about thinking and trying to gain understanding of concepts. There should be technology free components of assessment to assess thinking/reasoning skills” (Australia).

“For pre-college mathematics, computer adaptive testing is the only practical way to implement mastery learning, which I believe is essential for these students. For the most part I think technology in calculus courses should be confined to numerical methods, which is where I have my students use it” (USA).

Evidence of all of the five assessment issues depicted in Table 3 was found in the observational study of technology implementation at The University of Auckland. With respect to the first issue of congruency for instance, the foundation course on introductory calculus (Maths 102), which first introduced graphics calculators in 1997, allowed calculator use in all aspects of the course, with explicit study guide statements encouraging technology use in all areas of assessment [2, p. 215]. However, even during the height of the CAS-calculator period from 2001 to 2005, student ownership of CAS-calculators never exceeded 50% (often significantly less), which means that any intended congruency in final examinations was not in fact realised for the majority of

students. Such less than optimal outcomes were one of the factors supporting the change to a computer-based technology policy for the whole department in 2006, as explained in an interview by one of the key proponents of CAS-calculators:

The aim of using the calculators was that they could be used across the board in lectures, assignments, tests and exams. The difficulty is that the entire medium is in rapid revolution, and we only had partial adoption, the number of students who had the TI-89's was too small. The evolution of Matlab, with the symbolic toolbox, is cheaper for them to buy, and easily available in the labs, so lots more use it. [2, p. 223]

While the use of Matlab meant that students could no longer actively engage in a hands-on fashion with the technology in lectures or examinations, this was not actually a change for the majority of the students who did not possess calculators. The greater overall access to technology provided by the computer laboratories was seen by the above respondent as outweighing this particular drawback. Non-availability in examinations is also not necessarily seen as incongruent in some courses, particularly in applied mathematics, as one interview respondent explains:

I don't see it (non-availability) as a problem...we assess it [technology] in our assignments and tutorials [lab-based], and that's a much more appropriate place...Maybe that's where it's different [from pure maths], it's not there as a support for them. Its different things we're assessing, when we use technology in applied maths, it's not for doing something they could equally well do in another way. The stuff we assess in exams is different material, or different aspects of the same material. [2, p. 224]

This explanation demonstrates how technology is often viewed differently by applied mathematics lecturers, with the emphasis much more on its value as a computational tool, as opposed to symbolic manipulation features, which are frequently seen as less useful. The core first year calculus and algebra course for the majority of students, Maths 108, does attempt to address the disadvantage of no access to Matlab in examinations (the nature of Maths 108 content can be gauged from the questions in Figures 2 and 3 and Table 5). Matlab use is actively promoted in Maths 108 assignments, using investigations and questions that require Matlab to solve, and questions are included in tests and examinations that require students to interpret Matlab output. This requires students to engage with the technology, and helps signal that the use of technology is valued in the course. However, by necessity, the examination questions fall largely into two categories, those that directly test familiarity with Matlab use, for example recall-type questions of Matlab features, and static reproductions of Matlab output which students are asked to interpret. Two multiple-choice examples are provided in Figure 2 to demonstrate this, the first from the 2007 summer school mid-semester test, and the second from the 2007 end-of-semester two examination. The second question in Figure 2 addresses two learning objectives commonly supported in the literature; it attaches value to the use of technology, and it uses technology to provide inter-representational links and examine conceptual understanding [2, pp. 94-110; 115-117]. However, while 108 examinations contain many similar questions requiring students to interpret Matlab output mathematically, and then apply this in solving problems, the literature suggests that for most effective learning, students should be actively engaged with the technology [2, pp. 91-92, 125]. In addition, there is the danger that testing conceptual understanding at the same time as technological facility unnecessarily complicates the question, especially for weaker students [2, pp. 112-114]. Two of the interview subjects involved with teaching Maths 108 see value in including questions such as those shown in Figure 2 in tests and examinations, but both admitted that it is less than ideal. One was optimistic that developments in cheaper laptop technology would soon see many more students with access to these in lectures, and potentially in examinations (where permitted).

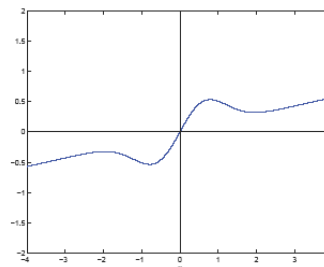
26. Which one of the following is a useful Matlab command for sketching an equation?

- (a) ezplot (b) drawit (c) plotit (d) ezdraw

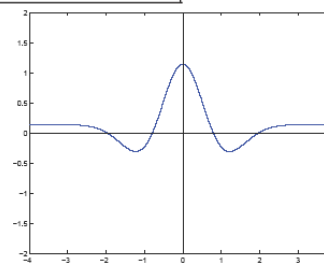
39. The Matlab output tells us that $f'(0.7967) = 0$. Which one of the following is TRUE?

- (a) Since $f(0.7967) = 0.5361 > 0$, the function f has a relative minimum at $x = 0.7967$
 (b) Since $f'(0.7967) = 0$, the function f is undefined at $x = 0.7967$
 (c) Since $f''(0.7967) = -1.4616 < 0$, the function f has a relative maximum at $x = 0.7967$
 (d) Since $f'(0) > 0$, the function f has a relative maximum at $x = 0.7967$

`>> ezplot(f)` produces the graph:



`>> ezplot(diff(f))` produces the graph:



`>> ezplot(diff(f,2))` produces the graph:

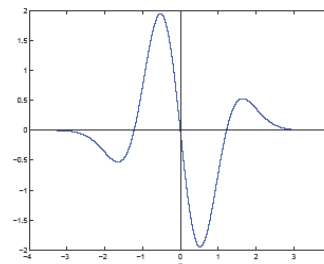


Figure 2 Sample questions from Maths 108 formal assessment [2, p. 225].

Some courses have pursued alternative forms of assessment, introducing online quizzes that examine basic skills and count towards final grades. Several smaller courses also conduct collaborative tutorials in the computer laboratories, where students are assessed using technology in a group environment. This is not a feasible option for the larger core courses (some with more than 900 students, taught in several large lecture streams). There is some evidence that alternative student solutions have been considered in the department, although not that widely and not always positively. The core course Maths 108 requires Matlab output for some assignment questions, with a small percentage of the final grade given specifically for Matlab worksheets. The multiple-choice format it uses for much of the test and examination questions also negates concerns about the need to recognise or accept alternative technology-inspired solutions, although this format was principally introduced for practical reasons (e.g. ease of marking and security against cheating). A mark-scheme from the foundation course Maths 102 in 2007 directs markers to accept correct solutions without working for many questions where technology may have been used to obtain the answer, but this practice is not favoured in many courses, particularly in examinations. For

example, the 2006 study guide for the first-year mathematics-major course Maths 150 emphasises full working, even if technology is used, while one interview respondent observed that more students are now giving “nonsense answers” in examinations, based on CAS-output, that show little comprehension of either the problem or the solution” [2, p. 226].

Observations suggest that questions asked in tests and examinations remain a significant issue in technology use: Are questions technology-active, -neutral, or -free; do the questions afford unfair advantages for students with access to technology; and what are the implications of fewer instrumental and more conceptual questions? Such issues were of considerable concern when graphics calculators were introduced to Maths 102, and Kiernan, Oates and Thomas [14] provide several examples in their report where it was found that the use of graphics calculators trivialised the question by providing direct solutions. Other questions gave significant advantages to students with access to calculators, who were able to view graphs that greatly enhanced their ability to understand the problem. The authors describe how, largely in the interests of equity, questions were re-phrased to neutralise such advantages as much as possible [14]. This practice is still maintained in Maths 102, as access to technology in examinations remains inequitably low.

While a formal, comprehensive analysis of Maths 108 questions has not been conducted, minutes from the department’s technology-committee meetings in 2005, and statements from interview respondents in the observational study, provide evidence that these issues have been considered. For example, four interview subjects all raised similar concerns about the nature of questions in a technological environment, although they differed somewhat in their views. They all noted that CAS-availability in examinations trivialises many of the common “padding-type” instrumental questions. One respondent [2, p. 226] described how in recent years, they have endeavoured to assign such questions to the web-based quizzes, while trying to ensure examination questions have a more conceptual basis. Another [2, p. 227] believes that routine skill-based questions are inappropriate in any case, so their removal due to technology would be a beneficial outcome. However, even with many routine questions removed, he still perceives that CAS-calculators offer an advantage, and he continues to emphasise them to students as essential in examinations.

Two studies [15, 16] suggest a means of inspecting the nature of examination questions, and identifying any changes which may have occurred in response to technology developments. Flynn and McCrae [16] compare three different classification schemes devised to assess the impact of CAS-calculators on examination questions. Impact is defined to mean that “a CAS-user would have access to a *more efficient* solution strategy than a graphics calculator user...not just a broader number of possible solution strategies” [16, p. 210]. The scheme by Kutzler [2000, in 16, p. 211], for example, identifies three categories of questions: *Primary* (CAS-use is the major activity); *Secondary* (CAS-use plays a minor role), and *No CAS use* (CAS is of no assistance). The scheme further differentiates within the first two categories, between questions which require superficial knowledge of the tool, and those that require sophisticated knowledge. However, Flynn and McCrae found all three schemes problematic to use, especially when differentiating between questions that require both conceptual understanding and algebraic manipulation:

...with any classification scheme, there is no clear-cut dividing line between the categories, because the reality is continuous, not discrete. Hence, for some exam questions it may appear arbitrary to put them in one or the other category. [Kokol-Voljc, 2000, in 16, p. 212]

Hong, Thomas & Kiernan [15] use a simpler classification to distinguish between *calculator-positive* and *calculator-neutral* questions, depending on whether the calculator provides any perceived advantage in answering the question. This evaluation does not require a judgement to be made about the effectiveness of the strategy, or the level of advantage it offers. Given the problems

Flynn and McCrae [16] encountered with the more elaborate schemes, and the fact that here we are primarily concerned with whether CAS-availability has had any effect on the nature of questions, as opposed to the degree of that advantage, the examples in this discussion are analysed using the simpler classification suggested by Hong, Thomas and Kiernan [15].

Three sample semester one Maths 108 examinations were selected for inspection, one for each distinct period of technology use (1999 pre-calculator; 2004 CAS-calculator and 2007 Matlab). Some elements of the comparison are not directly equivalent, for example the end-of-semester examination changed from three to two hours in 2006, multiple-choice questions were not introduced until 2004, and there were changes to content in 2006, such as moving eigenvalues and eigenvectors to the follow-on second-level course. The comparisons are therefore made using percentages of the total marks for which CAS was seen as advantageous. The consideration of whether CAS provides an advantage was done largely from the perspective of the TI-89 used most often by students in this course, but students are not limited in their choice of CAS; they can use higher-powered calculators such as the TI-92 or the newer TI-Nspire (the latter was not available at the time of this study), with additional advantages for some questions. The analysis was checked by a colleague who has taught Maths 108 and is familiar with the TI-89 and the TI-92. Unlike the study by Flynn and McCrae [16], providing an “advantage” was not limited necessarily to a more effective strategy, for example it includes here the checking of solutions, since this is a particularly useful, if somewhat time-consuming strategy for answering multi-choice questions. For several long answer questions, it was difficult to decide the exact proportion of marks, as some parts were aided by CAS, and others were not. Where a consensus could not be reached, the questions were left as undecided. The results are summarised in Table 4, which demonstrates a dramatic drop in CAS-positive questions from the short answer section in 1999, to the multiple choice section in 2004. Even with the subsequent increase in 2007, there is still a distinct drop from 1999. This reflects a response to the use of CAS-technology, with a move away from the mostly instrumental skills-based questions that are directly solvable using CAS, to more conceptually based CAS-neutral questions, as demonstrated later in the comparison between the first two examples in Table 5.

Table 4 Comparison of Maths 108 Examination Questions from 1999 to 2007

Year	Section of Exam: marks/total	Percentage of CAS-Positive Marks in this section.		
		TI-89	TI-92	Undecided
1999 (Before CAS-calculators)	Short Answers: 30/100	80	0	0
	Long Answers: 70/100	33	39	0
2004 (CAS-calculators)	Multiple Choice: 54/180	26	37	4
	Long Answers: 126/180	42	56	6
2007 (Matlab as principal technology, CAS-calculators allowed)	Multiple Choice: 40/120	35	0	0
	Long Answers: 80/120	40	0	8

The drop in questions favoured by the TI-92 in 2007 shown in Table 4 largely reflects the removal of skills-based questions on eigenvalues and eigenvectors, with this content being shifted to the next higher-level course Maths 208. This pattern, also seen in the long answer questions, suggests that access to higher-powered CAS technologies, such as those described by Flynn [16],

may afford greater advantages in courses where such advanced content is examined. The sample questions in Table 5 also demonstrate a change in the nature of technological advantages provided, with the trend towards fewer multiple-choice questions directly solvable using CAS. This is hardly surprising given that neither scientific nor graphics calculators were permitted in 1999. Consequently, there was no technological advantage, so no attention to this was necessary.

Table 5 Sample Examination Questions from Maths 108 (1999 to 2007)

Example and Year	Question
Example 1: 1999	If $f(x) = \frac{2x+5}{3x+1}$, find $f'(x)$
Example 2: 2007	When differentiating the following functions, for which is the Chain Rule useful? (a) $f_1(x) = \tan x \cdot \ln x$ (c) $f_3(x) = \tan x(\ln x)$ (b) $f_2(x) = \frac{\tan x}{\ln x}$ (d) $f_4(x) = e^x \tan x$
Example 3: 2007	Suppose it is known that $\int f(x)dx = e^x + C$. Then $\int f(x-1)dx =$ (a) $e^x + C$ (b) $e^{x-1} + C$ (c) $e^x - 1 + C$ (d) $e^x(x-1) + C$
Example 4: 2004	The function f , where $f(x) = \ln(\tan(x))$ has domain: (a) $(0,1)$ (b) $(-\infty, 0)$ (c) $(0, \infty)$ (d) $(1, \infty)$
Example 5: 2004	Given that x and y satisfy the equation $x^2 - y^2 = 2xy + 1$. One takes differentials. Which of the following is true? (a) The result is $2x - 2y(dx + dy) = 2$. (b) It is not possible to take differentials in this case. (c) The result is $2xdx - 2ydy = 2ydx + 2xdy$. (d) The result is $2dx - 2dy = 2dxdy$.
Example 6: 2007	Let $A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 4 \\ 6 & 6 \end{bmatrix}$. If $AB = C$, then which of the following represents the matrix B ? (a) $\begin{bmatrix} 4 & 4 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} -2 & -2 \\ 2 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
Example 7: 2007	$\int 12x^3 dx =$ (a) $3x^2 + C$ (b) $4x^2 + C$ (c) $3x^4 + C$ (d) $4x^4 + C$

The majority of the CAS-positive questions in later years resemble the third and fourth examples from Table 5, which evoke a congruency between pedagogy and assessment. While CAS does provide an advantage, these questions require a level of competency (instrumental genesis) with the use of the technology [3], combined with mathematical knowledge and conceptual understanding, in order for this advantage to be realised. One of the interview subjects [2, p. 229]

confirmed that a reduction in skills based questions, with greater attention to conceptual understanding, was an explicit objective of the Maths 108 teaching team as a result of the technology implementation. Example 5 in Table 5 illustrates the sophisticated CAS-knowledge required in some questions to realise an advantage [16]. While the TI-89 will provide a solution, considerable work is needed to recognise the solution as one of those presented in the choices provided. Example 6 is a question for which CAS gives a solution, but a competent student could solve this question much more efficiently without it. Such a question would not be considered CAS-positive using the scheme of Flynn and McCrae [16]. Example 7 shows that despite the big drop in numbers of such questions from 1999 to 2004, some questions are still trivial using CAS. These increased again from 2004 to 2007, although not to the 1999 levels, suggesting that less consideration has been given to calculator factors after the choice to adopt Matlab as the primary technology.

Unlike the short-answer sections, the figures for the long answer questions in Table 4 show a small increase in CAS-positive questions, which is consistent with considerations of congruency, and the objectives of the Maths 108 teaching team. While this seems a reasoned response to the introduction of CAS-calculators prior to the shift to Matlab, the comparatively low numbers of students with access to them raises concerns of fairness, and this is exacerbated in the Matlab era, since the percentage of CAS-positive questions has remained relatively constant, while the numbers of students with calculators has steadily declined. Like the multiple-choice questions, the majority of these require a level of conceptual understanding in order to gain any technological advantage, but they do still provide an advantage to students with CAS-access. For example, a question in the 2007 examination required students to find the derivatives of three separate functions, all of which were directly solvable using CAS, with minimal technical competency (low-level instrumentation). Given that the low proportion of students accessing the CAS-calculators was an important factor in the decision to change, the fact that examination questions continue to provide advantages to students with the calculators should be of great concern.

The long-answer questions which proved most difficult to categorise involved a combination of conceptual and technological objectives. Consider the example from the 2004 exam in Figure 3:

29.

The matrices $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 3 & -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 1 & 1 \\ -3 & 3 & 2 \\ 2 & -1 & -1 \end{bmatrix}$ are inverses of each other.

Use this fact to solve the following system of linear equations :

$$\begin{aligned} -x + y + z &= 4 \\ -3x + 3y + 2z &= 4 \\ 2x - y - z &= 1 \end{aligned}$$

Figure 3 Sample question from the 2004 Maths 108 examination.

Exactly how many of the five marks allocated to the above question could be considered CAS-positive could not be agreed upon by either analyst, even after a third opinion was sought from another colleague familiar with the course. While all three colleagues agreed that CAS provides a definite advantage in this question, even after assigning a step-by step marking schedule for the question, it was not obvious to what extent students were afforded this advantage. One felt that the main advantage lay in the ability to check the answers at each step, another felt that the

question required such a depth of understanding to know what to do, that the advantage gained in using the technology was largely instrumental.

4. Summary

The discussions in this paper demonstrate that, notwithstanding the holistic consideration of the taxonomy as advocated by Oates [2], assessment issues remained a significant individual factor in technology implementation at The University of Auckland. The impact of CAS on examination questions is seen as a particularly complex issue. Questions require real constant care and attention to balance the examination of students' skills against conceptual understanding in a fair and appropriate manner. Examiners should inspect questions from both a mathematical, and a CAS-perspective, such as the measure of CAS-positive questions used here [15]. They should also consider issues of equity associated with differences in the affordances provided between different forms of CAS, as certain CAS-products have been shown to provide significant benefits over others for some questions [16]. Even given the generally narrow technology choices adopted by students in the Auckland case study (Matlab and predominantly TI-89 CAS-calculators), considerable advantages were still apparent in some examination questions.

Oates [2, p. 253] emphasises assessment in one of six implications for integrated technology stemming from his study. He summarises the issues described in this discussion, when he concludes that “assessment issues remain problematic, even in an otherwise integrated environment...Continued vigilance is required to attend to the inequitable advantages afforded by unequal access to technology”, whether that be physically, through differing levels of student instrumentation, or the affordances provided by different types of technology.

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